

Answers to Problem Set 5

Total: 45 marks

6.14 [5 marks]

1. $\text{SameRow}(b, f) \vee \text{SameRow}(c, f) \vee \text{SameRow}(d, f)$
2. $\neg \text{SameRow}(c, f)$
3. $\text{FrontOf}(b, f)$
4. $\neg(\text{SameRow}(d, f) \wedge \text{Cube}(f))$
5. $\neg(\text{FrontOf}(b, f) \wedge \text{SameRow}(b, f))$
6. $\nabla \text{Cube}(f)$
7. $\nabla \text{SameRow}(b, f)$
8. $\text{FrontOf}(b, f) \wedge \text{SameRow}(b, f)$ ✓ ▽ \wedge Intro: 3,7
9. \perp ✓ ▽ \perp Intro: 8,5
10. $\nabla \text{SameRow}(c, f)$
11. \perp ✓ ▽ \perp Intro: 10,2
12. $\nabla \text{SameRow}(d, f)$
13. $\text{SameRow}(d, f) \wedge \text{Cube}(f)$ ✓ ▽ \wedge Intro: 12,6
14. \perp ✓ ▽ \perp Intro: 13,4
15. \perp ✓ ▽ \vee Elim: 7-9,10-11,12
16. $\neg \text{Cube}(f)$ ✓ ▽ \neg Intro: 6-15

6.20 [5 marks]

1. $A \vee B$	
2. $A \vee C$	
3. $\neg A$	
4. $A \vee (B \wedge C)$	✓ \vee Intro: 3
5. $\neg B$	
6. $\neg A$	
7. $A \vee (B \wedge C)$	✓ \vee Intro: 6
8. $\neg C$	
9. $B \wedge C$	✓ \wedge Intro: 8,5
10. $A \vee (B \wedge C)$	✓ \vee Intro: 9
11. $A \vee (B \wedge C)$	✓ \vee Elim: 6-7,8-10,2
12. $A \vee (B \wedge C)$	✓ \vee Elim: 3-4,5-11,1

6.25 [5 marks]

1. $\neg A \wedge \neg B$	
2. $\neg A \vee B$	
3. $\neg A$	
4. $\neg A$	✓ \wedge Elim: 1
5. \perp	✓ \perp Intro: 4,3
6. $\neg B$	
7. $\neg B$	✓ \wedge Elim: 1
8. \perp	✓ \perp Intro: 7,6
9. \perp	✓ \vee Elim: 2,3-5,6-8
10. $\neg(A \vee B)$	✓ \neg Intro: 2-9

6.31 [5 marks]

The argument is invalid, as the following world shows. (In this world b is a small dodec, but it could instead be a *medium cube*.)



T	1. $\text{Dodec}(b) \vee \text{Cube}(b)$
T	2. $\text{Small}(b) \vee \text{Medium}(b)$
T	3. $\neg(\text{Small}(b) \wedge \text{Cube}(b))$
F	4. $\text{Medium}(b) \wedge \text{Dodec}(b)$

6.32 [5 marks]

1. $\text{Dodec}(b) \vee \text{Cube}(b)$	
2. $\text{Small}(b) \vee \text{Medium}(b)$	
3. $\neg \text{Small}(b) \wedge \neg \text{Cube}(b)$	
4. $\text{Dodec}(b)$	
5. $\text{Small}(b)$	
6. $\neg \text{Small}(b)$	✓ \wedge Elim: 3
7. \perp	✓ \perp Intro: 5,6
8. $\text{Medium}(b)$	✓ \perp Elim: 7
9. $\text{Medium}(b)$	
10. $\text{Medium}(b)$	✓ Reit: 9
11. $\text{Medium}(b)$	✓ \vee Elim: 5-8,9-10,2
12. $\text{Medium}(b) \wedge \text{Dodec}(b)$	✓ \wedge Intro: 11,4
13. $\text{Cube}(b)$	
14. $\neg \text{Cube}(b)$	✓ \wedge Elim: 3
15. \perp	✓ \perp Intro: 14,13
16. $\text{Medium}(b) \wedge \text{Dodec}(b)$	✓ \perp Elim: 15
17. $\text{Medium}(b) \wedge \text{Dodec}(b)$	✓ \vee Elim: 4-12,13-16,1

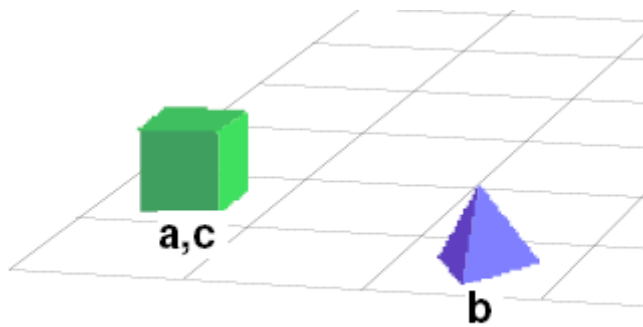
Extra [5 marks]

1. $A \vee D$	
2. $A \vee \neg D \vee E$	
3. $\neg E \vee \neg B$	
4. ∇A	
5. $A \vee \neg B$	✓ $\nabla \vee$ Intro: 4
6. ∇D	
7. ∇A	
8. $A \vee \neg B$	✓ $\nabla \vee$ Intro: 7
9. $\nabla \neg D$	
10. \perp	✓ $\nabla \perp$ Intro: 6,9
11. $A \vee \neg B$	✓ $\nabla \perp$ Elim: 10
12. ∇E	
13. $\nabla \neg E$	
14. \perp	✓ $\nabla \perp$ Intro: 13,12
15. $A \vee \neg B$	✓ $\nabla \perp$ Elim: 14
16. $\nabla \neg B$	
17. $A \vee \neg B$	✓ $\nabla \vee$ Intro: 16
18. $A \vee \neg B$	✓ $\nabla \vee$ Elim: 3,13-15,16-17
19. $A \vee \neg B$	✓ $\nabla \vee$ Elim: 2,7-8,9-11,12
20. $A \vee \neg B$	✓ $\nabla \vee$ Elim: 1,4-5,6-19

6.36 [5 marks]

1.	
2. $\nabla a = b \wedge b = c \wedge a \neq c$	
3. $a = b$	✓ $\nabla \wedge$ Elim: 2
4. $b = c$	✓ $\nabla \wedge$ Elim: 2
5. $a = c$	✓ $\nabla =$ Elim: 3,4
6. $a \neq c$	✓ $\nabla \wedge$ Elim: 2
7. \perp	✓ $\nabla \perp$ Intro: 5,6
8. $\neg(a = b \wedge b = c \wedge a \neq c)$	✓ $\nabla \neg$ Intro: 2-7

6.37 [5 marks]



F 1. $\neg(a \neq b \wedge b \neq c \wedge a = c)$

6.41 [5 marks]

1.	
2. $\neg((A \wedge B) \vee \neg A \vee \neg B)$	
3. $\neg A$	
4. $(A \wedge B) \vee \neg A \vee \neg B$	✓ \vee Intro: 3
5. \perp	✓ \perp Intro: 4,2
6. $\neg\neg A$	✓ \neg Intro: 3-5
7. A	✓ \neg Elim: 6
8. $\neg B$	
9. $(A \wedge B) \vee \neg A \vee \neg B$	✓ \vee Intro: 8
10. \perp	✓ \perp Intro: 9,2
11. $\neg\neg B$	✓ \neg Intro: 8-10
12. B	✓ \neg Elim: 11
13. $A \wedge B$	✓ \wedge Intro: 7,12
14. $(A \wedge B) \vee \neg A \vee \neg B$	✓ \vee Intro: 13
15. \perp	✓ \perp Intro: 14,2
16. $\neg\neg((A \wedge B) \vee \neg A \vee \neg B)$	✓ \neg Intro: 2-15
17. $(A \wedge B) \vee \neg A \vee \neg B$	✓ \neg Elim: 16